The Economic Meaning of the Sabbath and a Geometric Model of Debt

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Abstract. This paper explores the role of the Sabbath as an economic institution and its relationship with the concept of debt. The Sabbath's function is explored, both as a special day of abstinence from work, as well as a wider symbol of social liberation. The paper develops a geometric model that correlates the way the Sabbath commandment interacts with a law that institutes a de facto seven-year debt cycle. This shows that it is possible to use modern mathematics and economic concepts to model biblical laws that have economic content. It thereby shows a way to frame key biblical laws inside of a rational framework where they can be seen as a harmonic economic system that co-ordinates the real and financial sides of the economy. The key technical contribution of this paper is an application of Pythagoras' theorem to the modelling of the macroeconomics of debt.

Keywords: Commandments, Sabbath, debt, geometry, Pythagoras

1. Introduction

This paper uses the ideas of economic cycles and mathematical geometry to model the economic and mathematical content embodied within key biblical scriptures which relate to economics. The study of economic and financial cycles has long played a role in economics. Economic historians, for instance, have often used the concept of cycles to assess long and short swings in economic activity. These sorts of cycles have been associated with terms such as the Kuznets Swing, the Juglar cycle and the Kondratiev wave.

More modern macroeconomic literature also considers both business and financial cycles as well as their interactions. A range of studies find that the financial cycle is of longer duration than the business cycle (Drehman et al. 2012; Galati et al. 2016; and Herman et al. 2015). One study finds that "on average financial cycles last 7.2 years" (Schuler et al. 2015), whilst another finds the business cycle to be 6.3 years in duration (Herman et al. 2015). Whilst these are just two estimates within a large range of empirical values found in the literature, these sorts of numbers are not far away from comparative respective lengths of financial and non-financial cycles implied by considering biblical scriptures that relate to the concepts of the Sabbath and debt. The rest of the paper will firstly explore the concept of the Sabbath and its relationship with debt before developing a novel geometric model to replicate the

numerical and economic content found in biblical scriptures in relation to these same concepts.

2. The Sabbath

In the biblical cosmology, God established the seven-day week as a key unit of time, of which the Sabbath was its culmination. In this way, God instituted the Sabbath, which is an institution related to time which is in contrast to the physical elements of creation. Not only was the Sabbath a part of Creation but Isaiah 66:23 (New International Version, NIV) says that "From one New Moon to another and from one Sabbath to another, all mankind will come and bow down before me," says the Lord." This suggests that it will be an eternal institution. These biblical references are taken by the Seventh-day Adventist denomination, for instance, to be an argument for the enduring and perpetual relevance of the Sabbath in Christian religious practice. This paper supports this idea by identifying general properties of the Sabbath, as well as another key law related to debt; both of which are independent of any particular type of economy such as the agrarian-focused Ancient Israelite economy.

The Sabbath, whilst first appearing in Genesis, takes more of a central role in the bible in Exodus 20:8-11 (NIV), where in the fourth commandment of the decalogue, it says to

> Remember the Sabbath day by keeping it holy. Six days you shall labour and do all your work, but the seventh day is a sabbath to the LORD your God. On it you shall not do any work, neither you, nor your son or daughter, nor your male or female servant, nor your animals, nor any foreigner residing in your towns. For in six days the LORD made the heavens and the earth, the sea, and all that is in them, but he rested on the seventh day. Therefore, the LORD blessed the Sabbath day and made it holy.

This commandment describes the Sabbath as an institution that regulates economic activity. The Sabbath is, of course, much more than a form of economic regulation. It is a day that was made holy and was sanctified by God at the time of creation, and so serves as a central day of worship for Jews and some Christians such as Seventh-day Adventists. Heschel, for instance, describes the Sabbath as "a sanctuary which we build, a *sanctuary* in time" (Heschel 2005, 29). Nevertheless, the root-meaning of the word Sabbath is "to cease"; it is a day of rest where work was to cease along with associated buying and selling. From the vantage point of biblical cosmology, it can be seen that the "environment in which God placed humans is social" (Cafferky 2015, 36), and this is embodied by the institution of the Sabbath. It provides a general regulation on the extent to which economic agents could work or require others to work (including animals). There is also limited, if any, direct reference to any particular type of economy which further indicates a general applicability of this commandment.

The Sabbath commandment was the lynchpin of a series of laws that, at least in principle, engendered liberation across the social hierarchy within Ancient Israel. It was part of a body of stipulated economic cycles that also occurred over years and decades, culminating in the Jubilee. More generally, the concept of Sabbath rest is a motif that is woven throughout the Hebrew Scriptures and the New Testament. Whilst it refers to one day in a week, it also refers to one year out of seven. The concepts of sabbatical years (Exodus 23:10-1; Leviticus 25:4-5; 26:34-5; 2 Chronicles 36:21) and the year of Jubilee (Leviticus 25: 10-54; 27:17-24) present the Sabbath as a means of establishing liberation in a general sense. Moreover, in the New Testament, Christ made clear that the Sabbath was created for people, and was not intended to be a rules-based system of oppression (Mark 2:27-8) (Gallagher 2019, 141). There are also applications of the concepts of the Sabbath and debt forgiveness in the modern world. For instance, the economist Ched Myers (2023, 4) believes that:

> ... the theology of Sabbath economics, with its ethic of regular, systemic redistribution of wealth and power ... continues to offer communities of faith today a way out of our historical and persistent slavery to the Debt system – with its alienating cruel practices of social stratification and the concentration of wealth and power.

The Sabbath also references the specific inclusion of marginalized groups, such as foreigners, servants, and slaves, thereby highlighting the importance of social justice (Gallagher 2019, 138-40). All factors of production were to experience rest related to the number seven. For instance, the sabbatical principle of rest applied "as much to the earth as it [did] for humans" (Cafferky 2015, 36), as the land was to lie fallow every seventh year (Exodus 23:11).

Every seven days slaves would experience rest along with all of their respective masters' household, including animals. Then, every seventh year, slaves were to be set free, and debts were to be forgiven. This represented a release from responsibility of meeting financial debt obligations. So, whilst the Sabbath related specifically to the labor market, there was also another macroeconomic regulation related to the functioning of the credit market.

Moreover, slavery as reported in the bible generally emerged through a failure to pay debts. In general, slavery itself was a boundary solution to the problem of a person not being able to afford one's day-to-day needs. If daily needs could not be met, debt was the solution. If debts could not be repaid, slavery was a solution. As such, the seven-year release from debt obligations and the freeing of slaves can be viewed as being primarily all about debt, whether it was the forgiveness of debt directly or the release of fellow Israelites from debt-induced slavery.

A different version of the Sabbath commandment, found in Deuteronomy 5:12-5 (NIV), reads:

> Observe the Sabbath day by keeping it holy, as the LORD your God has commanded you. Six days you shall labour and do all your work, but the seventh day is a sabbath

> to the LORD your God. On it you shall not do any work, neither you, nor your son or daughter, nor your male or female servant, nor your ox, your donkey or any of your animals, nor any foreigner residing in your towns, so that your male and female servants may rest, as you do. Remember that you were slaves in Egypt and that the LORD your God brought you out of there with a mighty hand and an outstretched arm. Therefore the LORD your God has commanded you to observe the Sabbath day.

This version, referring to the slavery and oppression experienced by the children of Israel and the mixed multitude that accompanied them out of Egypt, provides a more direct link between the Sabbath, slavery, and debt. Here, the Sabbath commandment emphasizes that bondage was not to be a theme of God's economy. In this way we see that the Sabbath also engendered a moment of equality, whilst reflecting the freedom that God had granted upon freeing his people from the bondage experienced in the land of Egypt. Indeed, the Sabbath can be interpreted as intermittent moments of salvation. The weekly Sabbath represented a moment of freedom that mirrored the liberation from the Egyptians. It established periodic rest days where the master-servant bondage relationship was to be suspended and then re-established as the week reverted back to its first day. The Sabbath thereby reflects both the nature of liberation provided by Jesus Christ as well as the freeing of the Israelites from Egypt. This thereby links the creation story directly to the salvation provided by Christ via the institution of the Sabbath due to these periodic moments of liberation.

Consider also the Jubilee year, which occurred just after the 49th year of a 50-year cycle. It was to be a year of rest, a sort of super-sabbatical, where property was to revert to its original owners, and debts were to be forgiven (Anderson 2005, 27). The inherited land of families would also be returned if it had been sold due to debts that could not be repaid. In this way, the "jubilee year ... [affirmed] the fundamental unity of creation, since both people and the land [were] intended to observe the sabbath" (Fanucci 2014, 6). Overall, whilst the ratio of one-seventh was applied to both people and their land, the nature of the enforcement of the liberation was to vary in ways that aligned with the nature of the entity experiencing the liberation.

3. Debt, the Sabbath, and time ratios

The key scripture about financial market regulation is found in Deuteronomy, where it says the following:

> At the end of every seven years you must cancel debts. This is how it is to be done: Every creditor shall cancel any loan they have made to a fellow Israelite. They shall not require payment from anyone among their own people, because the Lord's time for canceling debts has been proclaimed (Deuteronomy 5:12-5, NIV).

This verse dictates that the debt cycle was to occur over seven years but at the end of the seventh year the debt cycle was to cease. More precisely, while debt contracts could be initiated by individuals at any time, these contracts (unlike the process of buying and selling) would not cease to exist every seventh day, but only at the end of the seventh year. Concurrently, six years of work (i.e., buying and selling) will have occurred in this time period, if the observance of the Sabbath commandment were to be simultaneously observed by the same society. The first key mathematical observation to be noted is that this implicitly leads to a ratio of time spent on the two most primary economic activities: buying and selling, and lending and borrowing. This ratio is 6:7, as six-sevenths of time would have been associated with buying and selling, whilst the full seven-year period would be associated with lending and borrowing. These laws can also therefore be interpreted as macroeconomic regulations of both the goods and credit markets. Moreover, the laws dictated two cycles were to run in harmony: a continuous cycle related to debt instruments being used (lending and borrowing) and a cycle of discrete activity, related to what we now might term as 'goods-market' activity (buying and selling).

We now develop a model that provides a novel geometric derivation of two economies: a non-debt economy versus a debt-based economy which aligns with the economics embodied in the Sabbath and debt laws referenced above. Our model shows that these laws can be viewed as regulating the interactions between non-debt-related activity (a No-debt economy) and debt-related activity in a temporal sense. It will be shown that the concept of a 'rest' for the agents exacting labor can be derived in a generalized depiction of a closed economy which has debt transactions, and that the time associated with this rest period in equilibrium is approximately one-seventh of the total time allocated to economic activity.

4. The model

The model assumes a closed economy, subsequently considering a scenario where debt is not an available instrument, and then comparing this scenario to scenarios where it is available. The model thereby identifies the contribution of the debt instrument to equilibrium output. This equilibrium result is then compared to the economics embodied in the Sabbath and debt laws.

It is assumed that a unit of output requires a unit of labor effort. In a No-debt economy (an economy with no access to debt instruments), the income levels of any agent over a given period cannot diverge from their respective expenditure levels. An economy with a debt instrument, however, allows income and expenditure levels to diverge.

We assume a finite length of discrete periods of equal length (i.e., years). We then consider each group and how they interact through their intertemporal budget constraints. The basis of this model is the familiar intertemporal budget constraint model found in standard macroeconomic textbooks. However, two amendments are made, one of which is standard and the other more novel. Regarding the non-novel amendment, the intertemporal

budget constraint is made into a finite-time intertemporal budget constraint. The second, more novel, amendment is two-fold: we partition the economy into two interacting groups of net creditors and net debtors, and then describe their interactions by depicting each group by a distinct intertemporal budget constraint. It is assumed that both intertemporal budget constraints are met with equality.

The equation below says that the discounted expenditure of either group, i, must be less than or equal to discounted income, which represents the basic idea of a discrete-time intertemporal budget constraint,

$$\sum_0^{\bar{T}} \frac{E_i(t)}{(1+r)^t} \leq \sum_0^{\bar{T}} \frac{I_i(t)}{(1+r)^t}, \qquad (1)$$

where E represents expenditure and I income. As this must hold for each group, i,the equation becomes the following:

$$\sum_0^{\bar{T}} \frac{E_i(t)}{(1+r)^t} = \sum_0^{\bar{T}} \frac{I_i(t)}{(1+r)^t}. \qquad (2)$$

Without loss of generality, let us assume that the interest rate is set to zero, thereby allowing us to ignore discounting. Over the full period $\bar{T}$, this means that total incomes must equal total expenditures for each group. However, at the halfway point $\frac{\bar{T}}{2}$, cumulative incomes up until this point need not equal expenditures. An agent's credit balance at the end of any period is the difference between the sum of their incomes and expenditures over that time. An agent is a creditor if the sum of their income in the first half of the total of all periods, $\bar{T}$, exceeds the sum of their respective expenditures. Similarly, a debtor is an agent whose expenditure exceeds their income over the first half of all periods, $\bar{T}$.

This leads to two central symmetries in the modelling approach used in this paper. Firstly, the total amount of expenditure in the economy equals the total amount of expenditure. Secondly, the difference between the incomes and expenditures of the Creditor group equals the negative of the difference between the incomes and expenditures of the Debtor group. In effect, the model in this paper presents two groups that act as if they are in a two-country world. We consider, according to this analogy, only those transactions that occur across borders. It is these transactions—and not the intra-country economic activity—that would impact international debt levels. This can be more clearly shown in a simple two-period discrete-time model of debt. In each period, the aggregate debt will relate only to the flows of economic activity between the groups.

Here are the two budget constraints, for the Debtor group (D) and Creditor group (C):

$$I_D^1 + \frac{I_D^2}{1+r} = E_D^1 + \frac{E_D^2}{1+r} \qquad (3)$$

$$E_C^1 + \frac{E_C^2}{1+r} = I_C^1 + \frac{I_C^2}{1+r}, \qquad (4)$$

where I represents income and E expenditure, and r is the intertemporal discount rate (or, the interest rate). We focus only on the flows between groups, as these flows are those that impact the debt level. We thus set intra-group economic activity to zero. Then, by symmetry, the following four equalities must hold: $I_D^1 = E_C^1$, $I_D^2 = E_C^2$, $E_D^1 = I_C^1$ and $E_D^2 = I_C^2$. Moreover, without loss of generality, we can set the interest rate, r, to be zero.

5. Visualizing a geometry

We now place this depiction of two interacting budget constraints on a two-dimensional cartesian plane. The x and y axes measure the incomes and expenditures of the Debtor and Creditor groups, respectively. The positive x axis measures the incomes of the Debtor group (D_{in}), and the positive y axis measures the income of the Creditor group (C_{in}). Likewise, the negative x axis measures the expenditures of the Debtor group (C_{ex}), and the negative y axis measures the expenditures of the Creditor group (D_{ex}). The axes are necessarily orthogonal because the respective incomes and expenditures of each group do not directly feed into the credit balances of the other.

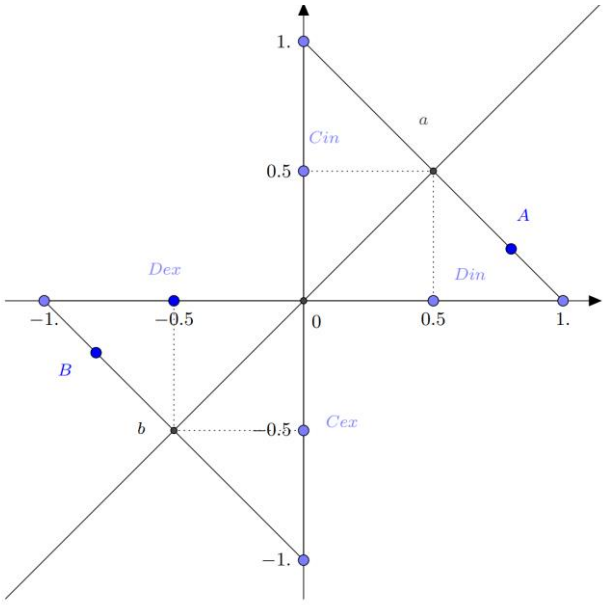

Figure 1. No-debt

Figure 1 represents an economy without a debt instrument. In this scenario, in absolute terms, the incomes (C_{in}) and expenditures (C_{ex}) of the Creditor group cannot diverge and will be equal to the absolute values of the incomes (D_{in}) and expenditures (D_{ex}) of the Debtor group, which likewise cannot diverge from one another. In this case, the two groups are undefined, which makes this is a limiting case. As incomes equal expenditures, and the axes are orthogonal, this can be represented geometrically by a 45-degree line going through the origin.

In all subsequent scenarios, debt will be used to fund expenditures. What will vary is the degree of divergence between the incomes and expenditures received and spent by each group. These will be characterized by tuples of activity, as the incomes of one group will necessarily be equivalent to the expenditures of another group, and they will meet orthogonally.

The next step is to alter the geometric shape of Figure 1 to reflect scenarios where expenditures are, to greater or lesser extent, funded by loans. The baseline case is where the Creditor group receives 100 percent of all incomes received by either group in the first period. This scenario is captured in Figure 2, which will be another baseline example, which depicts a re-distribution of total outputs as described in Figure 1, where all output is produced by one of the groups (the Creditor group), and where all purchases are financed by the other group (the Debtor group), via loans. This provides a baseline; all further scenarios where a debt instrument is available are then derived and measured in relation to this baseline example. All figures below now capture the first period in a two-period model, where each period is in fact a summation of a sequence of sub-periods.

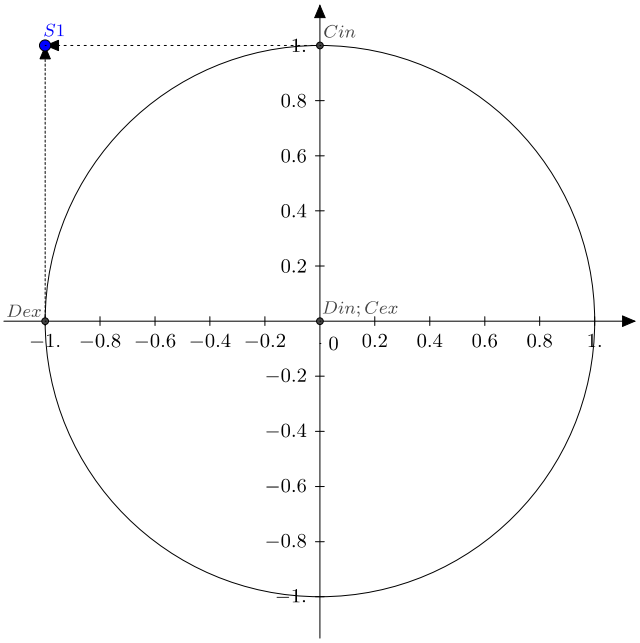

Figure 2. Maximum debt

Figure 2 shows a case where the Creditor group has zero expenditure, and the Debtor group has zero income with respect to 'between-group' transactions. This is described as 'Maximum debt,' where the Creditor group contributes one hundred percent of output and subsequently receives 100 percent of the income over the associated period of time. In this example, a debt instrument is both available and entirely necessary, as without it, zero percent of the economic activity that flows between groups would be realizable. The economy in Figure 2 is summarized by two points: (0, 1), the Creditor group's income, and (0, 0), the

Creditor group's aggregate expenditure. This shows that the Creditor group's expenditure is normalized to zero units and their income is normalized to one unit.

The key assumption of the model is that the scenario in Figure 2 is an accurate depiction of an equivalent economy to that depicted in Figure 1. The total area of the squares mapped out in the first and third quadrants of the unit circle sum to 0.5 of a unit squared in Figure 1. In contrast, the area of the square S_1 in Figure 2, which is comparable, is the size of a squared unit. Adding a debt instrument has thus doubled the area that captures the 'space' of economic interactions between the two groups (i.e., the potential pairings of units of incomes and expenditures flowing between the groups). This is intuitive, as debt allows for more potential interactions to occur.

We now use this baseline normalization to compare different alternative hypothetical realizations of the aggregate flows of incomes and expenditures. Another key assumption is that, when comparing scenarios from now on, the total area capturing the cartesian product of the potential between-group income and expenditure pairings is held constant. This makes intuitive sense, as it keeps the potential tuple pairings, captured by the squares in the second and fourth quadrants of the circle, henceforth S_1 and S_2, as partitions of a total potential 100 percent of between-group activity. This summarizes the potential different proportions of economic activity associated with each tuple of income and expenditure pairings. This captures how divergent the incomes and expenditures of the two groups are—and this is a key aspect of debt. It is, however, not the only important aspect of debt, as two incomes could be equal, but could both, at the same time, be funded by debt. This will be so in the equilibrium scenario. This approach ensures that each realization of the extent of divergence of the respective incomes and expenditures is comparable by keeping total area constant when comparing different percentages of expenditures associated with each group. However, debt instruments are assumed to be used to finance all purchases in the following scenarios of different debt-based economies.

Figure 3 below illustrates an intermediate case in which the respective income level of either group is also unequal, but not at the extreme as shown in Figure 2. The Creditor group receives more income than the Debtor group. In this case we keep the total size of the squares capturing the pairings of between-group transactions constant, at the size of a squared unit. Points E_1 and E_2 lie on the unit circle and will do so in each potential scenario capturing the percentage divergences of between-group output attributable to each group.

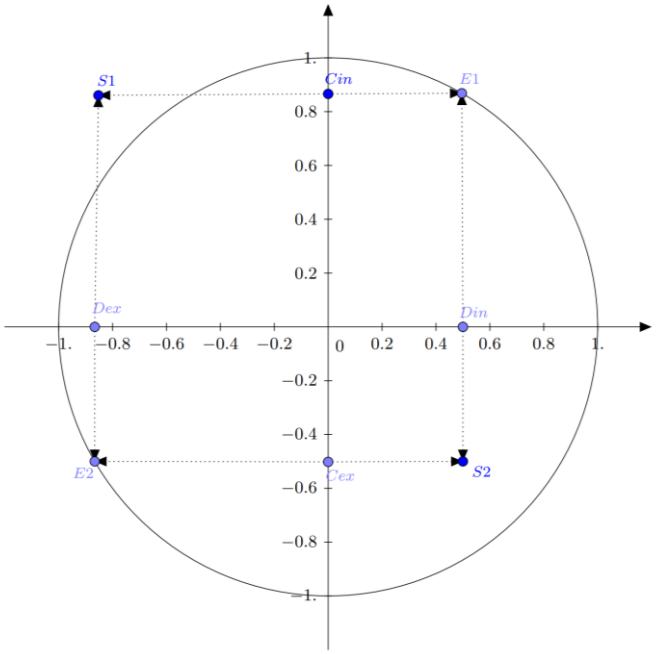

Figure 3. Intermediate debt

6. Pythagoras' theorem

Between examples, we have kept the total squared space of pairings of incomes and expenditures flowing between the group's constant. To compare alternative percentages, these would naturally relate to differing sizes of the two squares S_1 and S_2. Due to the orthogonality of the income and expenditure measures of the respective groups, a form of Pythagoras' Theorem holds, if we assume an inner product space (i.e., a Euclidean two-dimensional plane).

Proof

We first consider the Maximum debt example. This has a strong similarity to an economy with no debt instrument: instead of two agents producing a normalized level of 0.5, one agent in this scenario is accountable for 100 percent of output (a unit's worth of output) and the other, the Debtor group, spends this amount, being backed by a loan, and with no associated income.

Moreover, it has already been established that, in any closed economy, such as the world economy, aggregate expenditure must equal aggregate income. In this model, this is represented geometrically by a 45-degree line going through the origin. In a similar fashion, we can construct a line $y = -x$ going through the origin, which is associated with the square S_1 in the Maximum debt example of Figure 2. Hence, the Maximum debt economy is akin to a closed economy, with zero debt, where we have an economy characterized by a 45-degree line going through the origin, as in Figure 1. This represents an example in which 100 percent of

all income and output is associated with the Creditor group and 100 percent of all expenditure is associated with the Debtor group.

Expenditures and incomes that flow between groups are also assumed to meet orthogonally, and so squared space is used. This is because any incomes or expenditures of a particular group are associated solely with that group (i.e., each group has property rights over their respective incomes).

Let us now assume that incomes and expenditures of any closed economy are made up of the two elements u and v. In the Maximum debt example, for instance, we then have a square $S_1 = (u + v)^2 = 1$. This square characterizes the between-group transactions in the Maximum debt case. u can be taken to measure the Creditor group income received from the Debtor group's expenditures, which is equal in absolute value to the Debtor group's expenditure on the Creditor group's output. In contrast, v is taken to measure the Debtor group's income from the Creditor group, which is equal in absolute value to the Creditor group's expenditure on the debtor' group's output. In the case of Maximum debt, u equals 1, and v is zero.

We can now decompose the Maximum debt example square S_1 into two non-zero-sized squares S_1 and S_2, which capture varying proportions of total output associated with each group. These two squares will now capture the varying levels of imbalance between the relative incomes and outputs of each group; this yields the respective proportions of economic activity attributable to each group.

Let us define an Inner Product Space (a geometric space): u and $v \in$ V; where u and v are orthogonal and represent the incomes and expenditure levels of the two groups. We have

$$\| u \|^2 + \| v \|^2 = \| u + v \|^2 = 1 . \quad (5)$$

This means that there will be two squares that sum to the squared unit in each scenario. Now consider a finite length of time over which incomes and expenditures are measured. We let $\| u \|^2$ summarise the economic flows associated with the Creditor group's income and the Debtor group's expenditure, which are necessarily equal in absolute value and orthogonal. Likewise, we let $\| v \|^2$ summarize the flows of economic activity associated with the Creditor group's expenditure and the Debtor group's income. The sum of the two squares will always sum to the unit squared, as they jointly cover 100 percent of economic activity associated with debt-based economic activity. Expenditure in each case is assumed to be financed by debt, but the measure of divergence between the amounts of activity associated with each group varies between zero and 100. This shows that all potential percentage shares of economic activity flowing between the two groups, which relates to debt-related economic activity, can be described by the Pythagorean Theorem ■

The next section outlines a strong correlation between the equilibrium of this geometric model and the economic relationship between the Sabbath and debt laws discussed above.

7. Linking the geometric model to the biblical laws

Figure 4 shows the limiting case of the economy as the divergence between the income levels of both groups tends to zero, whilst all purchases are still funded by debt. It is a limiting case, because at this limit, neither group is defined. The Debtor group's income (D_{in}) must still equal the Creditor group's expenditure (C_{ex}). This captures the essence of an intertemporal budget constraint being satisfied by the end of a prescribed finite period, whilst allowing the incomes of the respective groups, debtors and creditors, to diverge in the interim (i.e., if the first period is actually a summation of a sequence of sub-periods). It captures an example where, even if all flows are funded by debt, there are an equal quantum of flows in either direction. This means that it is consistent both with the concept a No-debt economy and an economy that has a debt instrument. Hence, it is a type of equilibrium.

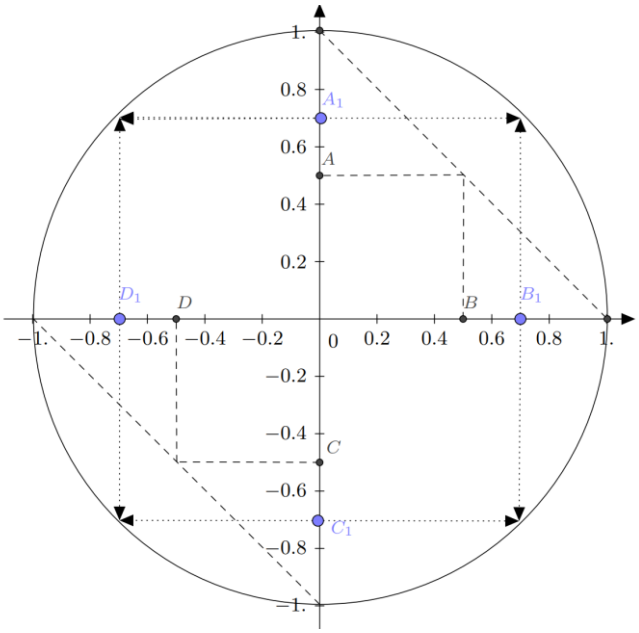

Figure 4. Equilibrium

This particular decomposition of the 'square' S_1, associated with the Maximum debt example of Figure 2, into two smaller squares, has those two squares being of equal size. This means that the overall economic activity flowing between the two groups happens to be consistent with a scenario where there is no debt instrument (i.e., Figure 1, as in both cases each group is responsible for an equal amount of total output). This is consistent with the satisfaction of the intertemporal budget constraints by a finite point in time, as each group accounts for exactly 50 percent of total economic output.

In this equilibrium scenario, debt is being used to co-ordinate activity that happens at different times. If there were no co-ordination problem, all the activity could in principle be funded without debt. In direct contrast, the economic activity in Figure 2 also requires a debt instrument and in the absence of a debt instrument, none of that economic activity would be realizable, irrespective of the timing of the economic outputs that were realized during the time period under consideration. In this equilibrium case, however, debt is available but is not necessary except as a means to address the co-ordination problem. In other words, debt is required to address a mis-sequencing of purchases and incomes with respect to the economic activity flowing between groups. If these purchases were to be sequenced so as to occur simultaneously, then debt would not be necessary.

We now relate this Figure 4 and Figure 1 to the juxtaposition of the Sabbath and debt laws. It will be shown that this debt-based-economy equilibrium case, when compared to the No-debt example shown in Figure 1, correlates with the content of the biblical laws.

As discussed earlier, biblical laws related to Sabbath and debt, when combined, determine a ratio of 6:7 when comparing the temporal space allotted to No-debt (buying and selling) activity with the space allotted to debt-based activity (lending and borrowing)[1]. This is because biblical laws dictate that the temporal space of the credit cycle is seven years, whilst if the Sabbath commandment is simultaneously observed, it means that only six out of those seven years involve work, and the associated buying and selling. Hence, over seven years there will be only six years associated with buying and selling, with one of the years spent in rest, which is equivalent to cessation from work.

It is now shown that a similar ratio has been derived geometrically. Firstly, note that the No-debt scenario, displayed in Figure 1, had an expected income level of both groups at a normalized level of (0.5, 0.5). In contrast, an equivalent scenario where debt instruments are used but outputs happen to also be equal, yields a point of $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$. Hence, the modelling approach has served to identify the impact on outputs attributable to the presence of a debt instrument being available to economic agents. This debt instrument facilitates economic activity that otherwise would not be realizable.

The point $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ is approximately (0.7, 0.7). At first glance, this seems to show that the No-debt economy requires a temporal space of approximately five-sevenths of that of an equilibrium debt-based economy. However, to obtain the correlation with the scriptural based derivation of a 6:7 ratio, we need to factor in a further consideration. To obtain the final

[1] It should be noted that this relies on the precise wording of the law about debt found in Deuteronomy 15:1. In the NIV text, quoted above, debts were to be cancelled at the end of every seven years, as opposed to sometime during or at the start of the seventh year. Verse 12 of Deuteronomy 15, NIV, reads: "If any of your people—Hebrew men or women—sell themselves to you and serve you six years, in the seventh year you must let them go free." This refers the release of slaves occurring 'within' as opposed to at the end of the seventh year. A simple comparison implies that the wording used in the first verse of the chapter is used deliberately to mean the very end of the seventh year as opposed to the intervening time-period (i.e., within that year). Moreover, the reference is to cancelling debts, with no mention of prohibiting new lending during the seventh year.

correlation between the ratios, one need only recognize that in the process of receiving a loan (as opposed to being a creditor), the Debtor group is in a state of de-facto 'rest' relative to the other group (i.e., the Debtor group rests, relative to the Creditor group). During that period, they receive a loan with which they can purchase output with no associated work effort. Let us now assume that the two groups are on average the same size. This is a reasonable assumption, as both groups produce the same amount of output in the equilibrium examples shown in Figures 1 and 4. As such, this is equivalent to assuming that any individual in the model has the same average output over the full debt cycle—which is consistent with the idea of all debts being repaid. Hence, whilst the presence of a debt instrument moves the economy from an equilibrium point of (0.5, 0.5) to approximately (0.7, 0.7), half of the economy (on average) will be de facto resting with respect to the other half. In the equilibrium case this is difficult to conceptualize, as both groups not only are using debt but also tend to have the same levels of income. However, within each period of a two-period model, it assumes that none of the purchases and sales of output are occurring simultaneously, and that they therefore need to be funded by debt. If the No-debt model scenario is running in parallel to the equilibrium scenario, this means that for economic activity up until the point (0.5, 0.5), both agents can be assumed to be working simultaneously with no relative rest occurring. However, for the additional output, of approximate value (0.2, 0.2), half of the economy (on average) will be resting in relation to the other half, which will be producing output. Thus, in the equilibrium case, when a purchase is being made, the group making that purchase by means of a loan is not putting forth effort and is in effect 'ceasing from work.' This 'cessation' from work needs to occur to facilitate debt occurring. In terms of the amount of labor effort expended in relation to each type of activity (no-debt versus debt-based economic activity), we therefore generate a ratio of 6:7 based on this geometrically derived model of economic activity.

It should be noted that both the scriptural derivation and the geometric derivation of the 6:7 ratio are general in nature, as nearly all economies employ the economic functions of buying and selling, and lending and borrowing.

8. Discussion: the wider context

This analysis begs the question of whether there is a 'normative' argument for Sabbath observance for any arbitrary society, given the 'positive' evidence related to the interactions of real and financial cycles. The analysis discussed above suggests that the Sabbath has a policy function of aligning two cycles of differing lengths: the real and financial cycles. The analysis indicates that the financial activity related to debt functions over a temporal cycle that is seven-sixths the length of real activity. According to this viewpoint of the weekly Sabbath institution, the Sabbath is a means of elongating the length of the real cycle by a ratio of seven-sixths, and thereby aligning it to coincide with the financial cycle related to debt-based transactions. Given that the analysis is related to general properties of any arbitrary

economy, these findings also indicate that this is a normative principle that could therefore be applied to any arbitrary economy and not only one that fits the profile of the agrarian Ancient Israelite economy. One caveat is that other laws related to the ownership of land suggest that the wider economic implications of biblical laws served to engender an agrarian economy, for instance through ensuring tribes and families had long-term rights to land to be used for agrarian purposes. Notwithstanding this, the Sabbath can be viewed as an institution that enhances social welfare by coordinating economic cycles. Given, however, that the framework used is not utilitarian, the present analysis is insufficient to support any idea of the Sabbath maximizing social welfare.

There are also archaeological reasons for accepting the general applicability of the biblical commandments insofar as the Israelite economy (which it may reasonably be argued that laws were intended specifically to cater for) was itself non-uniform. McNutt, for instance, references David Hopkins (1996), who emphasizes that there was not even "an ancient economy" in ancient Israel, but rather a multiplicity of economies that were induced by the 'complex and fragmented' geography. Two major economic zones are identified as being, firstly, rural areas and small villages and, secondly, a more fully developed and interregionally integrated town- or urban-based economy (McNutt 1999, 154).

This analysis, however, does presuppose that barter was not a central feature of the Israelite economy, or indeed economies in general. This is reasonable to suppose, as money is mentioned in the bible as early on as in Genesis. Genesis 17:23 (NIV), for instance, says: "On that very day Abraham took his son Ishmael and all those born in his household or bought with his money, every male in his household, and circumcised them, as God told him." The story of Joseph also mentions the use of money, where it says in Genesis 47:15 (NIV): "When the money of the people of Egypt and Canaan was gone, all Egypt came to Joseph and said, "Give us food. Why should we die before your eyes? Our money is all gone." A specific aspect of tithing laws also indicates a general usage of money in the Ancient Israelite economy. Deuteronomy 14:25 (New English Translation, NET): "you may convert the tithe into money, secure the money, and travel to the place the Lord your God chooses for himself." Furthermore, "money makes debt possible" (Graeber 2011, 21). This suggests that given that there was a law on debt in the bible, money must have played a sufficiently significant role in the Ancient Israelite economy for that law to make sense as an economic institution. Isaiah 24:2 (NIV) suggests that buying and selling, and lending and borrowing were considered typical economic activities in the Ancient Israelite economy, as they are in any modern economy. It reads: "it will be the same ... for seller as for buyer, for borrower as for lender, for debtor as for creditor."

Moreover, sociologists such as David Graeber and Caroline Humphrey argue that according to anthropological evidence, no barter economy ever existed anywhere. According to Humphrey, "no example of a barter economy, pure and simple, has ever been described, let alone the emergence from it of money; all available ethnography suggests there never has been such a thing" (Humphrey 1985, 48). Humphrey also has written that "There are few if

any whole economies of any sizable scale which are known to have operated by barter alone" (Humphrey & Hugh-Jones 1992, 6). Graeber describes the idea of barter systems being the forerunners of modern monetary economies as the "the founding myth of our system of relations" and that "there's no evidence that it ever happened, and an enormous amount of evidence suggesting that it did not" (Graeber 2011, 28).

More generally, Brueggemann presents the Sabbath as an economic system that contrasts with the system experienced within Egypt, with no significant reference to the specific realization of the Israelite economy, such as being predominantly agrarian. In referring to the Egyptian socio-economic system, he refers to it as being a system in which "there can be no Sabbath rest. There is no rest for Pharoah in his supervisory capacity, and he undoubtedly monitors daily production schedules ...; and of course there can be no rest for the slaves who must satisfy the taskmasters in order to meet Pharaoh's demanding quotas." In contrast the Sabbath based system was one there was no longer exploitation and the "Israelite economy is no longer determined and compelled by the insatiable production quotas of Egypt" (Brueggemann 1989, 3-5). Hence, although the territory into which the Israelites were to enter was agricultural in nature, its system was to be one where there would be "no permanent underclass," and the poor would not be seen as objects to be targeted through economic abuse but rather seen as neighbors to live in community with (Brueggemann 1989, 44).

Heschel describes the Sabbath as being independent of a particular economic context: "On the Sabbath we live, as it were, *independent of technical civilization*: we abstain primarily from any activity that aims at remaking or reshaping the things of space. Man's royal privilege to conquer nature is suspended on the seventh day" (Heschel 2005, 28-9). This presents a generalized view of the Sabbath, unconnected to a specific spatial context. Even writing in the context of the United States in the 1950s—an advanced modern non-agrarian economy—Heschel considered the Sabbath to still remain as "a concrete fact, a legal institution and a social order" (Heschel 2005, 16).

9. Conclusion

This paper has shown that the biblical Sabbatical cycle is more than merely a day of worship. It can be viewed as a key part of a form of macroeconomic regulation that spans time periods and types of factors of production. It can also be viewed as being part of a wider macroeconomic framework that encompassed the regulation of debt and bondage more generally. The Sabbath was a weekly moment of economic equality, where both the slave and slave master could be considered equals, as their slave-master relationship was suspended for a 24-hour period. It was part of a wider system that ensured liberation was an essential part of the ideal Israelite economy. Not only humans, but animals and land were incorporated in this regime of liberation. Moreover, the yearly cycles of Sabbatical rest, release and forgiveness bore a symmetric relationship to the weekly cycle of Sabbath rest. The Sabbath

precepts closely relate to biblical precepts on debt, and this relationship is modelled using the simple geometry of the Pythagorean theorem. This correlates well with an implied ratio of time to be associated with the economic activities of buying and selling, and lending and borrowing, which is a ratio of 6:7. Whilst Heschel (1989, 10) describes the meaning of the Sabbath in terms of a celebration of time rather than space, this paper models the economics of Sabbath commandment, in combination with a key debt law, using the abstract geometric spatial object of the circle.

References

Anderson, John. 2005. "A Biblical and Economic Analysis of Jubilee Property Provisions." *Faith & Economics* 46: 25–41.

Brueggemann, Walter. 1989. *Sabbath as Resistance: Saying No to the Culture of Now.* Westminster: John Knox.

Cafferky, M. E. 2015. "Sabbath: The Theological Roots of Sustainable Development." *Journal of Biblical Integration in Business* 18, no. 1: 35–47.

Drehmann, M., Borio, C., and K. Tsatsaronis. 2012. "Characterising the Financial Cycle: Don't Lose Sight of the Medium Term!" *BIS Working Papers No. 380.*

Fanucci, Laura K. 2014. "Release from the Slavery of Debt: The Jubilee Year for Ancient Israel and the Modern Global Economy." *Obsculta* 1, no. 1: 5–10.

Galati, Gabriele, Irma Hindrayanto, Siem Jan Koopman, and Marente Vlekke. 2016. "Measuring Financial Cycles in a Model-Based Analysis: Empirical Evidence for the United States and the Euro Area." *Tinbergen Institute Discussion Paper 16-029/III.*

Graeber, David. 2011. *Debt: The First 5,000 Years.* New York: Melville House.

Herman, A., D. Igan, and J. Sol. 2015. "The Macroeconomic Relevance of Credit Flows: An Exploration of U.S. data." *IMF Working Paper 15/143.*

Heschel, Abraham. 2005 [1951]. *The Sabbath.* New York: Farrar, Straus and Giroux.

Hopkins, David. 1996. "Bare Bones: Putting Flesh on the Economics of Ancient Israel." In *The Origins of the Ancient Israelite States*, edited by Volkmar Fritz & Philip R. Davies, 121–39. Sheffield: Sheffield Academic.

Humphrey, Caroline. 1985. "Barter and Economic Disintegration." *Man* 20, no. 1: 48–72.

Humphrey, Caroline, and Stephen Hugh-Jones. 1992. "Introduction: Barter, Exchange and Value." In *Barter, Exchange and Value: An Anthropological Approach*, edited by Caroline Humphrey & Stephen Hugh-Jones, 1–20. Cambridge: Cambridge University Press.

McNutt, Paula M. 1999. *Reconstructing the Society of Ancient Israel.* Westminster: John Knox.

Myers, Ched. 2023. *The Biblical Vision of Sabbath Economics.* Lab/ora Press.

Schuler, Y., P. Hiebert, and T. Peltonen. 2015. "Characterizing the Financial Cycle: A Multivariate and Time-Varying Approach." *ECB Working Paper Series No. 1846.*

Zerubavel, Eviatar. 1989. *The Seven Day Circle: The History and Meaning of the Week*. Chicago: University of Chicago Press.